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Developing science for result-based management in fisheries - a digression?

By, Mogens Schou
Ministry of Food, Copenhagen.
www.fvm.dk/yieldoffish. E-mail msc@fvm.dk; mobile +4522610575.

Abstract

Biological and economic fisheries science and advice are serving public management rather than private exploitation of fisheries resources. Management regulates the fisher's choice of methods, technology, area and level of activity with increasing detail thus fuelling the demand for such science. The main activity is aimed at reducing the gap between registered catches (landings) and total catches and at targeting larger fish. Management needs time to prepare and legislate and it regulates with rigid and uniform rules. This leaves little room for the industry to adapt to the dynamics of resource utilization opportunities. As a contrast, result-based management sets the targets (catch quotas, where all fish caught count on the fishers quota), and leave the choice of method in harvesting to the exploiter together with the obligation to fully document (by E-log and CCTV) that targets are observed. As EU considers result-based management with full catch accountability, science might look at the opportunities of such management. It is about comparing the ability of a result based management vis á vis a regulatory management to optimize food output in an environment where the ideal harvesting pattern is highly complex and variable. And it is about developing approaches and tools for the industry to continuously improve the result of the output. The paper points to possible science topics and areas that can contribute to the development and optimization of result based management, i.a. biological/hydrographic, economics/market, optimal management models and technology/methods

Keywords: Result based management; Catch-Quota Management (CQM); full documentation; EU fisheries policy; CFP.

Result based management for improved utilization of marine resources

1. What is result based management in fisheries

I base myself on the following conditions for result based management:

1. A management strategy to ensure achievement of clearly stated results that contribute to the optimal output/impact relation. In the case of fisheries ensuring MSY and the lowest environmental impact when taxing the commercial resources.

2. Output targets must be measurable, considered legitimate and relevant for the industry to optimize its efforts against. The utilization of commercial stocks is a clear candidate for a result based management as ”catch quotas” are defineable, measurable and easy to use as a production target.

3. Rules must be kept to a minimum leaving the industry with the field of opportunity and the incentive to innovate.

4. Documentation and monitoring to assess whether the result is within the targets and to generate data relevant to the continous setting and development of targets is crucial. In a result based management the exploiter must take responsibility for sufficient and correct data.

5. Why result based management? Basically the need for sustainable food production in a is better achieved by private ingenuity than by public regulation for 3 reasons. The industry is closer to the problem, the industry will develop methods and technology applicable to the situation while public management uses generally applicable rules irrespective of the concrete situation. The industry may change and develop methods and technology with short notice while public management needs time to assess, define, negotiate and legislate.

2. Introduction

Utilizing the marine food potential to its full requires a high degree of knowledge, skill and technology to overcome the variability and complexities in the harvesting.

Basically the present fisheries regulation is aimed at reducing the gap between registered catches (landings) and total catches and at targeting larger sized fish. To-day EU fisheries management focuses on managing the fishers use of methods, technology, fishing-area and level of activity in order to reduce the gap. A management by “command and control”. It is normally appreciated that wealth generation in our society is best served by market based approaches where the ingenuity and drive of the individual is allowed to unfold within certain boundaries ensuring the common interest. While a command and control approach define the acceptable actions of the individual a result based management set the boundaries and let the individual choose and develop his actions.

The nature of result based management in fisheries is that science and public management set the targets and refrains from regulating how the targets should be met. He who exploits the resource has the choice of method in harvesting and the obligation to document that targets are observed. Instead of fixing landing quotas the targets are set as “catch quotas”.

Fixing the targets for the individual fish stocks takes place in an ecosystem context where primary ecosystem factors such as trophic interaction enter into the equation. The results are catch-quotas for each individual stock or stock component. All catches including possible discards count against the catch quota. This is how I understand F_{MSY} implementation in its final form.

Secondary ecosystem considerations relate to non-commercial species such as marine mammals and to habitat protection. Regulation in this area may be result-based – e.g. a max quota for harbor porpoise. However, most likely result based management is not a broadly applicable option here for the time being. Especially habitat protection is most likely best served by horizontal rules as area closures and regulation of gear construction.

The picture as I see it is shown in the annexed sheet. It can be expressed with a fisher representatives words: 'go take the fish, and don't wreck the place'. My primary focus in this paper is to consider result-based management as a way to the optimal output from commercial stocks.

The objective of management: Optimal utilization of fish stocks can be compared to filling out the shape of an amoeba with squares. If they are too large they either extend beyond the shape or they don't fill the shape to its boundaries. If the squares are of sufficient smallness and number they may cover the amoeba. However as the amoeba changes shape a highly dynamic adaptation of the squares is needed. Management by command and control can be compared to the amoeba illustration where the optimal exploitation requires a considerable amount of rules to cover all the decisions relevant to the stock impact that the fisher may take. An improvement of the present unsatisfactory situation will require even more management, control and science input by a system now strained on all three parameters.

Three fundamental criticisms can be levied against management by command and control in a complex environment. It requires centralized knowledge of extreme quality and a regulatory framework with forbidding costs and demands for qualified administration. It does not take account of the effect on fishermen's earnings with an obvious conflict between objectives. And an amoeba changes shape by the instant while regulations are subject to knowledge gathering, negotiations and legal process – completely without the dynamics needed in competitive production processes in a changing environment.

In contrast, a result based management, where the fisher account for the output and not his choice of method will incentivize him to obtain the optimal output through continuous use and innovation of methods and technology. Catch-Quota Management (CQM) is a result based management with the following main features:

- The fisher is accountable for the result only. The 'result 'is the total catch quota he can take. All catches including by-catches must be covered by a quota and all catches of the given species, including eventual discards count on the quota.

- In the result based management the fisher decide how to obtain the result. There are no regulations regarding methods or technology.

- The result is obtained at sea where no efficient public control is possible. The fisher carries the responsibility to fully document that he stays within the quotas. He must do so reliably e.g. by CCTV and sensors. If he fails to do so he is ultimately out of the catch-quota management.

Seen from a public perspective the main points of interest are:

- Innovation, knowledge and investments will successively improve utilization with the effect that the 'amoeban area of opportunity' is gradually filled out.
- TAC/Quotas can be increased as there is no need to take account of discards on public level
- Data for model building and advice will improve rapidly in quality.
- Public management and control can be reduced
- Promoting fish can be done in good faith and with an increasing inclusion of information that consumers may reward.
- Result based management – not the least full documentation can produce instruments to serve the secondary ecosystem considerations too.

I see no fundamental criticism to be levied against CQM as a resource management system. However, some balances and challenges are crucial.

- CQM and full documentation requires a transformation of management methods and investments in electronic systems and traced distribution chains.

- When the quota for one species in a mixed fishery is exhausted the fisher must stop the mixed fishery. To-day he can discard non quota species and still make money from the target species. That must go. Will he be able to target the fishery and obtain quotas for his catches in order to catch the quota for all species in the mixed fisheries? I believe this is the case in the North Atlantic, where supposedly relative few "choke species" could block for the utilization of other species. Probably this is not at present the case in highly mixed fisheries as in the Mediterranean. In addition to planning and conducting a selective fishery, the allocation of quotas between fishermen is important. As all catches must be accounted for, a high degree of transferability of quota-rights is important. Market based catch-share systems have shown to be effective in facilitating this – if designed well.

- As mixed fisheries must stop when the least plentiful quota is exhausted it is the advice on species abundance for mixed stocks is accurate and up to date. Failure to take account of e.g. big recruiting year classes may result in inoptimal catch results.

I have omitted the political and legal considerations regarding the transformation from regulatory to result based management, and it should also be noticed that CQM entails a professionalization of fisheries which may put more pressure on small scale fishing if not counteracted.

3. Political background and present experience

Denmark, Germany, France and UK in March 2011 jointly pointed to result-based management by CQM as the basis for the revised CFP.

> *"We are convinced that genuine fully documented catch-quota systems will promote sustainable development and a sustainable approach to fishery management focussing on total removal" -Ministers' declaration 1st March 2011*

The proposal for a CQM management is backed by extensive practical trials since 2008; currently with app. 60 vessels in 5 countries participating in Catch-Quota Management (CQM) and full documentation (CCTV cameras and sensors). By the end of 2010 Denmark had performed 50.000 hours at sea with CQM and full documentation.

A workshop with the following points was held in Copenhagen 17th May 2011

- WS 1 A reliable data chain for management, control and advice
- WS 2. "Smart control": Innovating control i.a. by data combining or new technology
- WS 3. Cost efficiency of full documentation versus traditional control
- WS 4. Technical rules – can they be simplified under CQM

The results of the workshop, papers, press clips and videos regarding CQM can be seen at www.fvm.dk/yieldoffish .

The 13th July 2011 the EU Commission presented its proposal for a revised CFP [COM(2011) 425 Final]. The Commission i.a. propose full catch accountability with an obligation to land all fish, introduction of transferable catch concessions, full documentation to ensure all fish landed and an improved science base for management. The Commission has not, however included thoughts on the simplification of regulations and control that may flow from the proposal.

The political process in Parliament and Council should lead to an adoption of the revised CFP at the end of 2012.

The role of science

1. A new perspective

Biological and economic fisheries science and advice are serving public management rather than private exploitation of fisheries resources.

Science could address the issue of optimal potential marine production and ways to support this in a result based context characterized one the one hand by catch data that are fully reliable and on the other hand a drive for innovation that stems from the fishery itself. The following science areas could be considered.

1. Biological/hydrographic science could,
   - delimitate and develop relevant output factors that constitute the basis for result based management.
   - improve models and concrete advice on basis of real time fully documented date. Improve analysis by making use of the industry's interpretative participation in a process where the argument is not about the figures but the reason for the figures.
   - develop real time or floating quota allocation models instead of present calendar year TAC/quotas. Within the calendar year concept models for year to year quota flexibility could be considered.

2. Economic/social science could focus on the cost efficiency potential of result based management for society and for the individual fisher, and it could do so in a static or dynamic perspective where innovation and technology drivers ensure successive improvement of the impact/result relationship in fishing. So far the property right question has been exercised extensively while I find little work relating the amount of fish that we can get out of the sea, e.g. with a result based management.

3. Technological science, innovation and best cases decide the speed with which the industry is able to optimize the opportunities given to them, and smart documentation, traceability and informative marketing will set the agenda for fish as a trustworthy food – this issue being a serious barrier for the marketing of fish as a notoriously healthy food.

2. Possible concrete science topics

The following are some incomplete illustrations of areas where science and advice could contribute to sustainability and wealth generation in a result based management.

Biological/hydrographic

- The catch-quota may gradually as knowledge and skills develop, be defined with a higher degree of refinement, e.g. by fixing quotas for separate stock components with a view to further improving the utilization pattern and the long-term output, and reduce pressure on vulnerable components.
- Options and consequences of CQM in relation to more precise targeting of the individual age groups in a stock – e.g. by fishing on a variety of age groups to obtain a differentiated outtake. This may lead to a more optimal output pattern, and it may reduce fishery induced evolution of the species.
- Consequences of multiannual quota allocations – e.g. allowing a large recruiting year class to be caught the following year – or to give a fisher with an individual quota to postpone his harvesting until the fish has grown into the high priced market sizes.
- What is the potential output of commercial stocks? In the North Sea an improvement of 10%? 50%?

Economics, social science and the market

Most of the suggestions under biological/hydrographic have an economic dimension too:

- Management and control costs
- Costs and benefits in relation to fully documented and traced fishery products e.g. as the 'sustainability stamp' can be integrated in the process instead of being certified by third party involvement.
- Economic science could focus on the cost efficiency potential of result based management for society and for the individual fisher, and it could do so in a static or dynamic perspective where innovation and technology drivers ensure successive improvement of the economic result of fishing.

Optimal management

<u>Choke species</u>

CQM entails that a fishery require a catch quota for all commercial species caught in that fishery. When the least plentiful quota in a mixed fishery - the "choke species" - is exhausted, the fishery must stop. To-day app. 60 vessels in DK, UK and Germany fish under CQM (on a limited scale and so far only for cod) subject to this condition. The principle imply that it is better to lose some catch opportunities due to lack of quota than continue fishing and discarding the non-quota species. CQM will incentivize selective fishing and transferable quota systems. Good solutions to the "choke species problem" are essential for the economic result of the fishery. I am convinced that viable solutions will be gradually and quickly found in the North Atlantic while the situation in the mixed fisheries of e.g. the Mediterranean is different.

In the annex professor Henrik Holm has developed an algorithmic solution to the problem. His model will make it possible when fixing TAC/quotas to assess consequences for total quota uptake of varying relationships between the least and the most plentiful species in mixed fisheries, of changes in vessels targeting ability and of changes in quota transferability.

Analyses of management approaches that can handle the problem of "choke-species" would be relevant. Transferable-catch shares is one approach. The Danish catch share management includes privately managed pools. The pools manage a market based swapping and leasing system allowing fishermen to land fish for which they do not have a quota and to lease or swap the necessary fish afterwards. In the public management a pool is managed as one vessel allowing for great flexibility within the pool. Analyses of the pools have shown that the quotas available for swapping and leasing are plentiful. The pools have established a code that fishermen should not discard fish for non-quota reasons as long as the pool as such has quotas available. Still some upgrading may happen and the test of the flexibility should be seen in context of the CQM, where lack of quotas cannot be solved by discarding. See more at http://puljefiskeri.dk/

Other systems as risk pools have been established. See e.g. West Coast Trawlers' Network for a risk pool agreement.

The problem of choke species may also be considered in a trade-off-model where a defined "overfishing" for one species in a mixed fishery may be accepted in order to optimize the catch of the mixed fishery as such. Here MSY would apply on a mixed stock basis rather than on a single stock basis. Such an approach should be used with caution as it would reduce the incentive to refine the precise targeting of the individual stock or to implement a flexible quota allocation.

Management based on overall quotas or year class specific quotas

Is the optimal biological and economic output of a stock linked to a given minimum size of the fish caught? Year after year? And should management care whether the stipulated "result" in the form of a quota is taken as fish in market size 5 (the smallest) or market size 1?

From a biological point of view I suggest that the optimal outtake of each year class from a given stock will vary from year to year depending on the actual relative strength of the year classes of the given stock. Why not supplement the advice on TACs' with a recommended "year class specific" TAC? and why not tell how the optimal outtake of each year class may affect the overall TAC that can be taken?

From an economic point of view a differentiated approach might also be interesting. However the adopted MSY principle work with tonnes not value.

From an administrative point it is quite feasible to count a vessel's catches against its quota with a correction factor based on the market size of the fish caught. If in this way for example 25% of a fleet switched from trawl to gill netting the average size of the fish would increase and the fishing mortality decrease considerably.

In a result based management it is possible to define the result on a year class level. Clearly it is possible to disincentivise catches of juveniles by a high deterrence factor. The problem of fixing the proper incentive however, may show that we try to steer a very big ship with a very small rudder. This remains to be seen.

From a genetic point of view there is little doubt that fishing over a larger range of year classes will reduce the fishery induced evolution.

Technology and methods

CQM will induce user driven innovation in selective fishing methods to ensure that only the marketable fish are caught. Thus management should now focus on under water effects not accounted for, negative effects on habitats, destroying of fish not being caught and quality deterioration on fish being caught.

- Pre-catch identification of quantity, size distribution and species composition.)
- Active selectivity and release in fishing gears
- Low impact -> high quality fishing

The above research area is currently being explored in CRISP (Centre for Research-based Innovation in Sustainable fish capture and Pre-processing technology)

Commercial use for non-targeted fish

If CQM is followed by a landing obligation ("discard ban") it will result in species and sizes of fish not previously being brought to habour must be considered as a new resource. Product development, marketing etc are required to ensure that these landings of fish become an economic benefit rather than a cost related to fishing.

New Fisheries Management - aligning economic forces for conservation

New Fisheries Management address the objectives of optimal food output, ecosystem considerations and allocation of rights to ensure the resource is used with a balanced fleet capacity.

1. Catch-Quota Management (CQM) is a result based management. Fishermen account for the result— the total outtake, of their fishery. CQM ensures the optimal utilization of commercial stocks available. The model is operational as is demonstrated by DK and UK fishing trials since 2008. Full documentation is an essential element of CQM.

2. Taxing of the resource must take place in respect of the ecosystem. 3 considerations are counted 1) The trophic dynamics are integrated in the MSY set TAC/quotas 2) Reduced catch of non-target species; 3) Protection of habitats and environment.

3. Allocation of user rights must be efficient in the senses that they balance catches with the fleet in the short term and fleet with catch opportunities in the long term. Transferability or flexible allocation of user rights is essential.

	Go, take the fish	And	Don't wreck the place
Objective	Maximizing the food output from commercial fish stocks by MSY. That is: TAC/Quotas based on carrying capacity of the individual stock and on direct ecosystem effects (trophic interaction)		Avoid and minimize unwanted effects of fishing
Strategy	Result based management of output: (the total catch) -> Catch-Quota Management for full accountability Market based management of the rights to catch -> quota transferability to align catch with fleet		• Result based management if possible (e.g. harbour porpoise) • Credit schemes where science is week and species inextricable • Generally applicable rules to protect habitats etc
Effects	Catch quota management with full documentation will: • exchange incentives to discard with incentives to selective fishing and new uses for 'all fish landed'. Ease a discard ban • remove the need for detailed regulation and control • Give precise data • ensure compliance and market access Catch Share management with high transferability will: • enable fleet/quota balance on long term • allow quota adaptations to mixed species catches (e.g. Danish pool system and EU in-year transfer scheme)		Do we have a coherent management that optimizes the food production base in respect of the ecosystem? More at www.fvm.dk/yieldoffish

Optimization in Catch Quota Management

Asst. Prof. Henrik Holm

Department of Basic Sciences and Environment
Faculty of Life Sciences
University of Copenhagen
Thorvaldsensvej 40
DK-1871 Frederiksberg C
Denmark
E-mail: `hholm@life.ku.dk`
URL: `http://www.matdat.life.ku.dk/~hholm/`

In a mixed fishery (we consider in this paper only mixed fisheries involving two different types of fish) managed by *catch* quotas (as opposed to *landing* quotas), a vessel must stop fishing as soon as one of the quotas in its quota portfolio has been reached. A vessel may use its equipment to influence its catch composition within a certain range, and thereby try to optimize its catch in order to maximize the income (market prices for the types of fish involved in the mixed fishery are given). A priori, a vessel's quota portfolio is given, however, it is possible to lease quotas from other vessels in order to obtain a more desirable portfolio.

Therefore, a vessel fishing under catch quota management has several ways to optimize its catch and income. We shall give precise mathematical formulations of the relevant optimization problems—and solve them. We illustrate with a number of concrete examples.

1 The Single Vessel Case

In this section, we consider a fishery in which a single vessel[1] catches two types of fish, called *type 1* and *type 2*, in some combination. We assume in this section that the vessel is completely isolated in the sense that fish quotas cannot be leased or bought from others vessels. We shall get rid of this restrictive assumption in Section 2.

Example 1.1 Some typical type 1/type 2 fish combinations are:

- cod/haddock (Scottish fleet)
- cod/nephrops (Danish fleet)
- sole/plaice (Dutch fleet)

Our goal in this section is, in a nutshell, to determine the type 1/type 2 catch composition that results in the maximal income given that the vessel's targetting ability may not allow it to fully utilize both catch quotas in the mixed fishery. To find the optimal catch composition, we must first give a mathematical description of the situation we are considering.

[1] The single vessel under consideration could be a specific vessel in some fleet, however, it could also be an "abstract" vessel representing a group of actual vessels fishing under similar conditions. Another possibility is to think of the vessel as an average of an entire fleet.

1.1 Assumptions

The market prices (kr/kg or kilokr/ton) of the two types of fish determine the market data:

Market Data	
Market price of type 1 fish	p_1
Market price of type 2 fish	p_2

Example 1.2 Prices obtained for landed catches vary a lot. Suggestive levels are:

$$\begin{aligned} p_{\text{cod}} &= 20 \text{ (kilokr/ton)} \\ p_{\text{haddock}} &= 14 \text{ (kilokr/ton)} \\ p_{\text{nephrops}} &= 55 \text{ (kilokr/ton)} \\ p_{\text{sole}} &= 85 \text{ (kilokr/ton)} \\ p_{\text{plaice}} &= 10 \text{ (kilokr/ton)} \end{aligned}$$

The vessel under consideration catches fish of type 1 and type 2 in some combination determined by the *catch composition parameter*, $c \in [0,1]$, whose value the vessel can partially influence by using various types of equipment. The catch composition parameter is simply defined as *the percentage of type 2 fish in the total catch*. Thus we have:

$$\begin{aligned} &\text{The percentage of type 1 fish in the total catch is} &:& \quad 1-c \\ &\text{The percentage of type 2 fish in the total catch is} &:& \quad c \end{aligned}$$

Example 1.3 The value $c = 0.55$ corresponds to a type 1/type 2 catch composition of $45\%/55\%$.

The vessel's fishing equipment allows it to vary the catch composition parameter in a certain range from a *lower catch composition bound*, l, to an *upper catch composition bound*, u.

Example 1.4 The values $l = 0.5$ and $u = 0.6$ correspond to the situation where the vessel may vary its type 1/type 2 catch composition in the range from $50\%/50\%$ to $40\%/60\%$.

For the vessel in question, we assume that the bounds l and u are given. The value of the catch composition parameter $c \in [l,u]$ is then to be determined such that the vessel maximizes its income. The vessel's fishery is bounded by its *type 1 fish quota*, Q_1, and its *type 2 fish quota*, Q_2 (in tonnes). In *catch quota management*, the vessel must stop fishing once it reaches one of these two quotas.

Example 1.5 Examples of member state quotas (in tonnes) are:

$$\begin{aligned} &\text{Scottish fleet:} & Q_{\text{cod}} &= 11000 & \text{and} && Q_{\text{haddock}} &= 21000 \\ &\text{Danish fleet:} & Q_{\text{cod}} &= 4000 & \text{and} && Q_{\text{nephrops}} &= 4000 \\ &\text{Dutch fleet:} & Q_{\text{sole}} &= 10000 & \text{and} && Q_{\text{plaice}} &= 20000 \end{aligned}$$

Thus, a single vessel in the Scottish fleet may have quotas $Q_{\text{cod}} = 11$ and $Q_{\text{haddock}} = 21$ (tonnes).

We summarize the mathematical data describing the vessel under consideration:

Vessel Data	
Type 1 fish quota	Q_1
Type 2 fish quota	Q_2
Lower catch composition bound	l
Upper catch composition bound	u
Catch composition parameter	c

Here Q_1, Q_2, l, and u are given, whereas c is to be (optimally) determined in the range $[l,u]$.

1.2 The Mathematical Model

Denote by t_1 and t_2 the number of tonnes of fish of type 1 and type 2 caught by the vessel. In catch quota management, both of these numbers are bounded by the vessel's given quotas, that is,

$$t_1 \leqslant Q_1 \tag{i}$$

$$t_2 \leqslant Q_2 \tag{ii}$$

The total catch is $t_1 + t_2$ of which t_2, by definition of the catch composition parameter, constitutes the fraction c, that is,

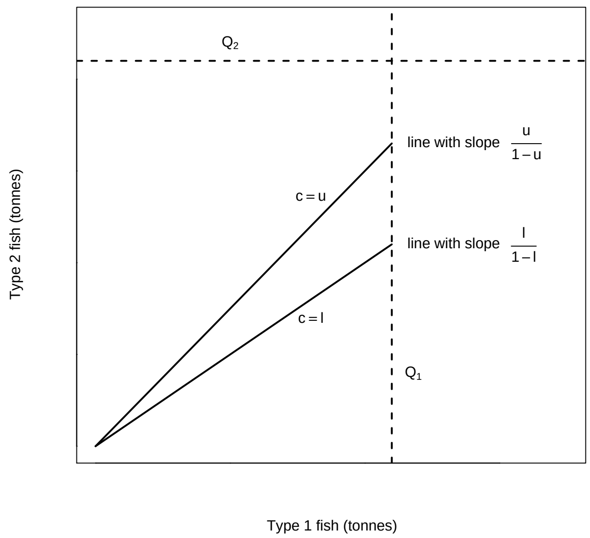
$$\frac{t_2}{t_1 + t_2} = c.$$

Equivalently,

$$ct_1 - (1-c)t_2 = 0 \tag{iii$_c$}$$

This is a straight line in a (t_1,t_2) coordinate system with slope $a = \frac{c}{1-c}$ and constant term $b = 0$. Since c can vary in the interval $[l,u]$, the slope of the line in (iii$_c$) can vary in the interval $[\frac{l}{1-l}, \frac{u}{1-u}]$.

The restrictions (i), (ii), and (iii$_c$) are illustrated in the following (t_1,t_2) coordinate system.



The vessel's *income* for catching (and selling) t_1 tonnes of fish of type 1, and t_2 tonnes of fish of type 2 is computed from the market prices:

$$I(t_1,t_2) = p_1t_1 + p_2t_2.$$

For each value of the parameter c in the interval $[l,u]$—corresponding to a choice of type 1/type 2 catch composition within the range determined by the vessel's equipment—the maximal income is found by optimizing the income function $I(t_1,t_2)$ (in two variables) subject to the boundary conditions (i), (ii), and (iii$_c$). This optimization problem, in which the third boundary condition depends on the value of c, may be written in short form as follows.

$$\begin{cases} I(t_1,t_2) = p_1t_1 + p_2t_2 = \text{Max!} \\ t_1 \leqslant Q_1 \\ t_2 \leqslant Q_2 \\ ct_1 - (1-c)t_2 = 0 \end{cases} \tag{P_c}$$

The optimization problem (P_c) above is an example of a so-called *linear programming* problem in two variables (t_1,t_2), which—given any specific value of c—is easily solved.

Example 1.6 We consider a whitefish fishery where type 1/type 2 fish are cod/haddock. Prices and quotas are as in Examples 1.2 and 1.5, that is,

$$p_1 = 20 \qquad Q_1 = 11$$
$$p_2 = 14 \qquad Q_2 = 21$$

Suppose, as in Example 1.3, that the vessel aims for cod/haddock catch composition of 45%/55% corresponding to the catch composition parameter $c = 0.55$. To maximize its income, the vessel must solve:

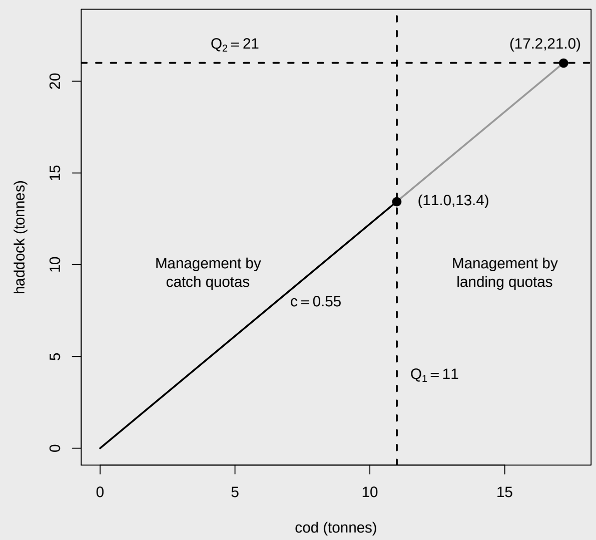
$$\begin{cases} I(t_1,t_2) = 20t_1 + 14t_2 = \text{Max!} \\ t_1 \leqslant 11 \\ t_2 \leqslant 21 \\ 0.55t_1 - 0.45t_2 = 0 \end{cases} \tag{$P_{0.55}$}$$

As illustrated below, the solution to this problem is:

$$(t_1^*,t_2^*) \simeq (11, 13.4) \qquad \text{(tonnes).}$$

Note that the ratio of $t_2^* = 13.4$ and the total catch $t_1^* + t_2^* = 24.4$ really is $c = 0.55$, and that the fishery must stop since the cod quota $Q_1 = 11$ has been reached. Therefore, subject to management by *catch quotas*, the income is:

$$I^* = I(t_1^*,t_2^*) = I(11,13.4) = 20\cdot 11 + 14\cdot 13.4 \simeq 408 \ \text{(kilokr).}$$



In comparison, the present management by *landing quotas* would generate an income of

$$I(11,21) = 20 \cdot 11 + 14 \cdot 21 = 514 \text{ (kilokr)},$$

that is, 106 (kilokr) more than under catch quota management. However, the resulting discards of cods would be $17.2 - 11 = 6.2$ (tonnes), which has a market value of 124 (kilokr).

Thus, under management by landing quotas—and with the given assumptions—one must, in some sense, "throw away" 124 (kilokr) to earn an extra 106 (kilokr).

Recall that our goal is to find the optimal catch composition, that is, the catch composition which results in the maximal income. In view of this, the example above, although illustrative, is not very informative since it requires the catch composition parameter c to be known in advance. To overcome this obstacle, we return to the optimization problem (P_c). For any given c in $[l,u]$, we denote by

$$(t_1^*(c), t_2^*(c))$$

the *maximum point* of the problem (P_c), and we let

$$I^*(c) = I(t_1^*(c), t_2^*(c)) = p_1 t_1^*(c) + p_2 t_2^*(c)$$

be the associated *maximal income*.

Example 1.7 Consider the setup of Example 1.6. As seen in loc. cit., one has in this case:

$$(t_1^*(0.55), t_2^*(0.55)) = (11, 13.4) \quad \text{and} \quad I^*(0.55) = 408.$$

With this notation at hand, we can give a precise mathematical formulation of the vessel's income-optimization problem.

The Single Vessel Optimization Problem. The following data are given:

- The market prices p_1 and p_2.
- The vessel's quotas Q_1 and Q_2.
- The vessel's lower and upper catch composition bounds l and u.

To maximize its income, the vessel must find the value $c^\circ \in [l,u]$ of the catch composition parameter which makes the income $I^*(c)$ as large as possible, that is,

$$I^*(c^\circ) = \max_{c \in [l,u]} I^*(c).$$

This *maximal income* is denoted by I°, and c° is called the *optimal catch composition parameter*. The corresponding *optimal (type 1, type 2) catch composition*, that is, $(t_1^*(c^\circ), t_2^*(c^\circ))$ is denoted by (t_1°, t_2°).

1.3 Solution

It takes only straightforward graphical considerations to solve the single vessel optimization problem formulated above. The precise solution presented below, which may seem a bit technical at first sight, will be important in Section 2.

Solution of the Single Vessel Optimization Problem. Depending on the three cases,

$$\begin{array}{lc} \text{Case I:} & \dfrac{Q_2}{Q_1} < \dfrac{l}{1-l} \\ \text{Case II:} & \dfrac{l}{1-l} \leqslant \dfrac{Q_2}{Q_1} \leqslant \dfrac{u}{1-u} \\ \text{Case III:} & \dfrac{u}{1-u} < \dfrac{Q_2}{Q_1} \end{array}$$

the optimal catch composition parameter c°, the optimal catch composition (t_1°, t_2°), and the maximal income I° are given as follows.

$$c^\circ = \begin{cases} l & \text{(case I)} \\ \dfrac{Q_2}{Q_1 + Q_2} & \text{(case II)} \\ u & \text{(case III)} \end{cases}$$

$$(t_1^\circ, t_2^\circ) = \begin{cases} (\frac{1-l}{l} Q_2, Q_2) & \text{(case I)} \\ (Q_1, Q_2) & \text{(case II)} \\ (Q_1, \frac{u}{1-u} Q_1) & \text{(case III)} \end{cases}$$

$$I^\circ = \begin{cases} (p_1 \frac{1-l}{l} + p_2) Q_2 & \text{(case I)} \\ p_1 Q_1 + p_2 Q_2 & \text{(case II)} \\ (p_1 + p_2 \frac{u}{1-u}) Q_1 & \text{(case III)} \end{cases}$$

Example 1.8 Consider the setup in Example 1.6, that is, type 1/type 2 is cod/haddock, and

$$\begin{array}{ll} p_1 = 20 & Q_1 = 11 \\ p_2 = 14 & Q_2 = 21 \end{array}$$

Assume that the vessel's equipment allows it to vary its cod/haddock catch composition in the range from 50%/50% to 40%/60%. As in Example 1.4, this corresponds to the bounds:

$$l = 0.5 \quad \text{and} \quad u = 0.6$$

The three cases I, II, and III described above are determind by the numbers

$$\frac{l}{1-l} = \frac{0.5}{1-0.5} = 1.0 \quad \text{and} \quad \frac{u}{1-u} = \frac{0.6}{1-0.6} = 1.5$$

Since $\frac{Q_2}{Q_1} = \frac{21}{11} \simeq 1.9 > 1.5$ we are in Case III. Hence, the formulae above give the optimal values:

$$c^\circ = u = 0.6 \quad , \quad (t_1^\circ, t_2^\circ) = (11, 16.5) \quad , \quad I^\circ = 451.$$

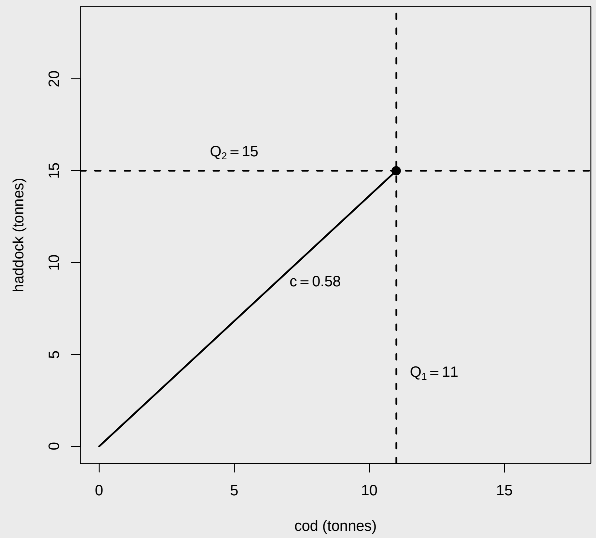
Thus, within the possible range, it is optimal for the vessel to aim for an cod/haddock catch composition of 40%/60%, in which case the fishery stops after having caught $t_1^\circ = 11$ tonnes of cod and $t_2^\circ = 16.5$ tonnes of haddock. The total market value for these fish are $I^\circ = 451$ kilokr.

We note that the catch composition 45%/55% chosen in Example 1.6 is *not* optimal since it only generates an income of 408 (kilokr).

Suppose that next season, the vessel's haddock quota drops from $Q_2 = 21$ to $Q_2 = 15$, say. The vessel must then change its fishery accordingly, indeed, now $\frac{Q_2}{Q_1} = \frac{15}{11} \simeq 1.4$ lies between 1.0 and 1.5 which puts us in Case II. This time, the formulae above give the optimal values:

$$c^\circ = 0.58 \quad , \quad (t_1^\circ, t_2^\circ) = (11, 15) \quad , \quad I^\circ = 430.$$

Hence, it is now optimal for the vessel to aim for an cod/haddock catch composition of 42%/58%, in which case both quotas $Q_1 = 11$ and $Q_2 = 15$ are reached *simultaneously*.



As implied by the example above, the formulae in our solution of the single vessel optimization problem can also be used to assess the boundaries for the quota relation between type 1 and type 2 fish which will make it possible for the vessel to obtain full quota utilization of both species

2 The Multiple Vessel Case

In this section, we expand on the situation from Section 1 by considering a mixed fishery in which *two* vessels[2], called *vessel A* and *vessel B*, both catch type 1 and type 2 fish in varying combinations. The vessels may may lease[3] quotas from each other in order to optimize their quota portfolios and, in turn, their income. From a mathematical point of view, the leasing aspect complicates the situation considerably compared to the single vessel case studied in Section 1.

[2] The two vessels under consideration could very well be two specific vessels in some fleet. Alternatively, one might think of the two vessels as being "abstract" ones representing some relevant situation. For example, one vessel could represent an actual group of vessels in some fleet fishing under similar conditions, while the other vessel could represent the "residual fleet", that is, the average of the remaining vessels in the fleet.

[3] In real life fishery, quotas may transferred (bought or leased) in some member states. *Buying* of quotas is understood as permanent buying of quota shares, and consequently the quota amount released every year on account of *total allowable catches* (TACs) is being set accordingly. Buying typically takes place in context of structural changes, as private scrapping investments in new vessels etc. Buying is not considered here. *Leasing* of quotas relate to the amount in tonnes of a given stock leased for the given quota year. Leasing may take place in context of planning the fishery for the entire quota year, or it may relate to daily adaptation of vessel quotas to the development in the fishery—or to cover unforeseen bycatches. The leasing element is considered here as an important tool of flexible quota management.

2.1 Assumptions

In the present situation, the market is defined by the market prices (in kr/kg or kilokr/ton) of the two types of fish, and by the leasing prices (also in kr/kg or kilokr/ton) of the two types of fish quota.

Market Data	
Market price of type 1 fish	p_1
Market price of type 2 fish	p_2
Leasing price of type 1 fish quota	q_1
Leasing price of type 2 fish quota	q_2

Example 2.1 Prices for leasing of quotas vary a lot. Suggestive levels are:

$$\begin{aligned} q_{\text{cod}} &= 10 \text{ (kilokr/ton)} \\ q_{\text{haddock}} &= 3 \text{ (kilokr/ton)} \\ q_{\text{nephrops}} &= 3 \text{ (kilokr/ton)} \\ q_{\text{sole}} &= 6 \text{ (kilokr/ton)} \\ q_{\text{plaice}} &= 2 \text{ (kilokr/ton)} \end{aligned}$$

Much of the data defining the two vessels under consideration are the same as described in Subsection 1.1, however, each vessel has its own quotas, its own catch composition bounds etc.

Vessel A Data	
Type 1 fish quota	Q_1^{A}
Type 2 fish quota	Q_2^{A}
Lower catch composition bound	l_{A}
Upper catch composition bound	u_{A}
Catch composition parameter	c_{A}
Type 1 fish quota leased from vessel B	x_1
Type 2 fish quota leased from vessel B	x_2

Vessel B Data	
Type 1 fish quota	Q_1^{B}
Type 2 fish quota	Q_2^{B}
Lower catch composition bound	l_{B}
Upper catch composition bound	u_{B}
Catch composition parameter	c_{B}
Type 1 fish quota leased from vessel A	$-x_1$
Type 2 fish quota leased from vessel A	$-x_2$

Since the two vessels may lease quotas from each other, we must consider two additional variables, x_1 and x_2. Here x_1 and x_2 are the number (in tonnes) of type 1 and type 2 fish quotas, respectively, which *vessel A leases from vessel B*. Alternatively, vessel B leases $-x_1$ and $-x_2$ tonnes of type 1 and type 2 fish quota from vessel A.

The pair (x_1, x_2) is called a *quota leasing agreement* between the vessels A and B.

Example 2.2 The situation where:

- vessel A leases 10 tonnes of type 1 fish quota *from* vessel B, and
- vessel A leases 15 tonnes of type 2 fish quota *to* vessel B

corresponds to the quota leasing agreement $(x_1, x_2) = (10, -15)$.

2.2 The Mathematical Model

Since vessel A can lease at most Q_1^{B} tonnes of type 1 fish quota *from* vessel B one has $x_1 \leqslant Q_1^{\text{B}}$; and since vessel A can lease at most Q_1^{A} tonnes of type 1 fish quota *to* vessel B one has $-x_1 \leqslant Q_1^{\text{A}}$.

Combining this with similar considerations for the variable x_2 give the following restrictions:

$$-Q_1^{\text{A}} \leqslant x_1 \leqslant Q_1^{\text{B}}$$
$$-Q_2^{\text{A}} \leqslant x_2 \leqslant Q_2^{\text{B}}$$

Once a quota leasing agreement (x_1, x_2) has been made, the vessels' quota portfolios change as follows:

- Vessel A's quota portfolio changes from $(Q_1^{\text{A}}, Q_2^{\text{A}})$ to $(Q_1^{\text{A}} + x_1, Q_2^{\text{A}} + x_2)$, and
- Vessel B's quota portfolio changes from $(Q_1^{\text{B}}, Q_2^{\text{B}})$ to $(Q_1^{\text{B}} - x_1, Q_2^{\text{B}} - x_2)$.

As in Subsection 1.2, we denote by $(t_1^{\text{A}}, t_2^{\text{A}})$ the number of tonnes of fish of (type 1, type 2) caught by vessel A. Similarly $(t_1^{\text{B}}, t_2^{\text{B}})$ denotes the number of tonnes of fish of (type 1, type 2) caught by vessel B. As described in Subsection 1.2, the catches for vessels A and B are subject to the conditions:

$$\begin{aligned} t_1^{\text{A}} &\leqslant Q_1^{\text{A}} + x_1 \\ t_2^{\text{A}} &\leqslant Q_2^{\text{A}} + x_2 \\ c_{\text{A}} t_1^{\text{A}} - (1 - c_{\text{A}}) t_2^{\text{A}} &= 0 \end{aligned} \qquad \text{and} \qquad \begin{aligned} t_1^{\text{B}} &\leqslant Q_1^{\text{B}} - x_1 \\ t_2^{\text{B}} &\leqslant Q_2^{\text{B}} - x_2 \\ c_{\text{B}} t_1^{\text{B}} - (1 - c_{\text{B}}) t_2^{\text{B}} &= 0 \end{aligned}$$

Since it costs money to lease quotas from the other vessel (dually, one gets paid for leasing quotas to the other vessel), the income functions for the two vessels become more complicated, in fact, they are given by:

$$I_{\text{A}}(t_1^{\text{A}}, t_2^{\text{A}}, x_1, x_2) = p_1 t_1^{\text{A}} + p_2 t_2^{\text{A}} - q_1 x_1 - q_2 x_2,$$
$$I_{\text{B}}(t_1^{\text{B}}, t_2^{\text{B}}, x_1, x_2) = p_1 t_1^{\text{B}} + p_2 t_2^{\text{B}} + q_1 x_1 + q_2 x_2.$$

We begin by focusing on vessel A. The situation viewed from vessel B's point of view can be described similarly.

For each value of the parameter $c_{\text{A}} \in [l_{\text{A}}, u_{\text{A}}]$—corresponding to a choice of type 1/type 2 catch composition within the range determined by vessel A's equipment—the maximal income for vessel A is found by optimizing the income function $I_{\text{A}}(t_1^{\text{A}}, t_2^{\text{A}}, x_1, x_2)$ (in four variables) subject to the boundary conditions above, that is,

$$\begin{cases} I_{\text{A}}(t_1^{\text{A}}, t_2^{\text{A}}, x_1, x_2) = p_1 t_1^{\text{A}} + p_2 t_2^{\text{A}} - q_1 x_1 - q_2 x_2 = \text{Max!} \\ t_1^{\text{A}} \leqslant Q_1^{\text{A}} + x_1 \\ t_2^{\text{A}} \leqslant Q_2^{\text{A}} + x_2 \\ c_{\text{A}} t_1^{\text{A}} - (1 - c_{\text{A}}) t_2^{\text{A}} = 0 \\ -Q_1^{\text{A}} \leqslant x_1 \leqslant Q_1^{\text{B}} \\ -Q_2^{\text{A}} \leqslant x_2 \leqslant Q_2^{\text{B}} \end{cases} \qquad (R_{c_{\text{A}}}^{\text{A}})$$

If the parameter c_{A} is given then $(R_{c_{\text{A}}}^{\text{A}})$ is a so-called linear programming problem in four variables $(t_1^{\text{A}}, t_2^{\text{A}}, x_1, x_2)$, which can be solved using Dantzig's simplex algorithm. However, this is not particularly useful since we do not know the optimal catch composition parameter in advance. To overcome this difficulty, we note that for any *fixed* choice of quota leasing agreement $(x_1, x_2) \in [-Q_1^{\text{A}}, Q_1^{\text{B}}] \times [-Q_2^{\text{A}}, Q_2^{\text{B}}]$, the problem $(R_{c_{\text{A}}}^{\text{A}})$ reduces to a problem in two variables t_1^{A} and t_2^{A}, namely:

$$\begin{cases} I_{\text{A}}(x_1, x_2)(t_1^{\text{A}}, t_2^{\text{A}}) = p_1 t_1^{\text{A}} + p_2 t_2^{\text{A}} - q_1 x_1 - q_2 x_2 = \text{Max!} \\ t_1^{\text{A}} \leqslant Q_1^{\text{A}} + x_1 \\ t_2^{\text{A}} \leqslant Q_2^{\text{A}} + x_2 \\ c_{\text{A}} t_1^{\text{A}} - (1 - c_{\text{A}}) t_2^{\text{A}} = 0 \end{cases}$$

Note that this is nothing but the single vessel optimization problem which was formulated in Subsection 1.2 (the only difference being that Q_i^{A} is replaced by $Q_i^{\text{A}}+x_i$) and solved in 1.3. Thus, supposing that vessel A and vessel B make the quota leasing agreement (x_1,x_2), then vessel A *knows* how to fish optimally and thus maximize its income. In fact, the following formulae are immediate from our solution of the single vessel optimization problem.

Quota Leasing Agreements from the Viewpoint of Vessel A.

Suppose that vessel A and vessel B make the quota leasing agreement (x_1,x_2), changing vessel A's quota portfolio from $(Q_1^{\text{A}},Q_2^{\text{A}})$ to $(Q_1^{\text{A}}+x_1,Q_2^{\text{A}}+x_2)$. Vessel A then knows

- Its optimal catch composition parameter c_{A}°,
- The corresponding optimal catch, $((t_1^{\text{A}})^{\circ},(t_2^{\text{A}})^{\circ})$, and
- The associated maximal income I_{A}°.

In fact, depending on the three cases,

$$\begin{aligned}
&\text{Case I:} && \frac{Q_2^{\text{A}}+x_2}{Q_1^{\text{A}}+x_1} < \frac{l_{\text{A}}}{1-l_{\text{A}}}\\
&\text{Case II:} && \frac{l_{\text{A}}}{1-l_{\text{A}}} \leqslant \frac{Q_2^{\text{A}}+x_2}{Q_1^{\text{A}}+x_1} \leqslant \frac{u_{\text{A}}}{1-u_{\text{A}}}\\
&\text{Case III:} && \frac{u_{\text{A}}}{1-u_{\text{A}}} < \frac{Q_2^{\text{A}}+x_2}{Q_1^{\text{A}}+x_1}
\end{aligned}$$

one has the following formulae:

$$c_{\text{A}}^{\circ} = \begin{cases} l_{\text{A}} & \text{(case I)}\\ \dfrac{Q_2^{\text{A}}+x_2}{Q_1^{\text{A}}+x_1+Q_2^{\text{A}}+x_2} & \text{(case II)}\\ u_{\text{A}} & \text{(case III)} \end{cases}$$

$$((t_1^{\text{A}})^{\circ},(t_2^{\text{A}})^{\circ}) = \begin{cases} \big(\frac{1-l_{\text{A}}}{l_{\text{A}}}(Q_2^{\text{A}}+x_2),Q_2^{\text{A}}+x_2\big) & \text{(case I)}\\ (Q_1^{\text{A}}+x_1,Q_2^{\text{A}}+x_2) & \text{(case II)}\\ \big(Q_1^{\text{A}}+x_1,\frac{u_{\text{A}}}{1-u_{\text{A}}}(Q_1^{\text{A}}+x_1)\big) & \text{(case III)} \end{cases}$$

$$I_{\text{A}}^{\circ} = \begin{cases} \big(p_1\frac{1-l_{\text{A}}}{l_{\text{A}}}+p_2\big)(Q_2^{\text{A}}+x_2)-q_1x_1-q_2x_2 & \text{(case I)}\\ p_1(Q_1^{\text{A}}+x_1)+p_2(Q_2^{\text{A}}+x_2)-q_1x_1-q_2x_2 & \text{(case II)}\\ \big(p_1+p_2\frac{u_{\text{A}}}{1-u_{\text{A}}}\big)(Q_1^{\text{A}}+x_1)-q_1x_1-q_2x_2 & \text{(case III)} \end{cases}$$

Note that c_{A}°, $(t_1^{\text{A}})^{\circ}$, $(t_2^{\text{A}})^{\circ}$, and I_{A}° are functions of the variables (x_1,x_2).

Hence, for any choice of quota leasing agreement (x_1,x_2), vessel A's new quota portfolio $(Q_1^{\text{A}}+x_1,Q_2^{\text{A}}+x_2)$ results in the maximal income $I_{\text{A}}^{\circ}(x_1,x_2)$—provided, of course, that it fishes optimally, i.e. according to the optimal catch composition parameter $c_{\text{A}}^{\circ}(x_1,x_2)$. Thus, seen from the viewpoint of vessel A, the optimal quota leasing agreement (x_1,x_2) is the one that maximizes the function $I_{\text{A}}^{\circ}(x_1,x_2)$.

We shall return to the following example throughout the rest of this paper.

Example 2.3 The market for cods and haddocks are assumed as follows.

Market Data	
Cod market price	$p_1 = 20$
Haddock market price	$p_2 = 14$
Cod quota leasing price	$q_1 = 10$
Haddock quota leasing price	$q_2 = 3$

The Scottish fleet's total cod and haddock quotas are 11000 and 21000 tonnes, respectively. Suppose that half of the fleet (vessel A) can vary its cod/haddock catch combination in the range from 50%/50% to 40%/60%, and that the other half (vessel B) in the range from 80%/20% to 20%/80%. Thus one has:

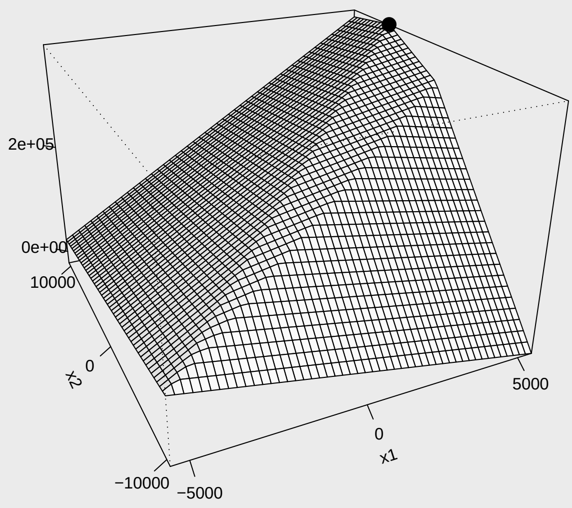
Vessel A Data	
Cod quota	$Q_1^{\mathrm{A}} = 5500$
Haddock quota	$Q_2^{\mathrm{A}} = 10500$
Lower catch composition bound	$l_{\mathrm{A}} = 0.5$
Upper catch composition bound	$u_{\mathrm{A}} = 0.6$

Vessel B Data	
Cod quota	$Q_1^{\mathrm{B}} = 5500$
Haddock quota	$Q_2^{\mathrm{B}} = 10500$
Lower catch composition bound	$l_{\mathrm{B}} = 0.2$
Upper catch composition bound	$u_{\mathrm{B}} = 0.8$

If the vessels choose not to lease quotas from each other—corresponding to the quota leasing agreement $(x_1, x_2) = (0, 0)$—then we may consider them as isolated. In this case, our solution to the single vessel problem in Subsection 1.3 shows that their maximal incomes are:

$$I_{\mathrm{A}}^{\circ} = 225500 \qquad \text{and} \qquad I_{\mathrm{B}}^{\circ} = 257000$$

As seen above, the given constants $Q_1^{\mathrm{A}}, Q_2^{\mathrm{A}}, Q_1^{\mathrm{B}}, Q_2^{\mathrm{B}}, l_{\mathrm{A}}, u_{\mathrm{A}}$ completely describe vessel A's maximal income $I_{\mathrm{A}}^{\circ}(x_1, x_2)$ as a function of the quota leasing agreement (x_1, x_2) made by vessels A and B. Its graph is:



Graph of income function $I_{\mathrm{A}}^{\circ}(x_1, x_2)$ for vessel A

It follows that the optimal quota leasing agreement *for vessel A* is $(x_1^{\circ}, x_2^{\circ}) = (5500, 6000)$ (meaning that vessel A should try to lease 5500 tonnes of cod quota and 6000 tonnes of haddock quota from vessel B) resulting in the new quota portfolio $(Q_1^{\mathrm{A}} + x_1, Q_2^{\mathrm{A}} + x_2) = (11000, 16500)$ and a larger maximal income:

$$I_{\mathrm{A}}^{\circ}(5500, 6000) = 378000.$$

A couple of observations are in order here:

If vessel A were given the quota portfolio $(11000, 16500)$ to begin with, its maximal income would have been 451000, cf. Example 1.8. In the present situation, vessel A must pay $5500 \cdot 10 + 6000 \cdot 3 = 73000$ to obtain the quota portfolio, thus reducing its maximal income to $451000 - 73000 = 378000$.

To obtain the larger (than 225500) maximal income of 378000, it is required that vessel A fishes optimally, that is, according to the optimal catch composition parameter $c_A^\circ(5500, 6000) = 0.6$, corresponding to a cod/haddock catch composition of 40%/60%. In this case, vessel A's fishery stops after having caught

$$(t_1^A)^\circ(5500, 6000) = 11000 \text{ tonnes of cod, and}$$
$$(t_2^A)^\circ(5500, 6000) = 16500 \text{ tonnes of haddock,}$$

that is, both quotas are reached simultaneously.

Of course, the quota leasing agreement $(x_1^\circ, x_2^\circ) = (5500, 6000)$, which is optimal for vessel A, can never become a reality since it reduces vessel B's cod quota to zero and thus stops vessel B's fishery.

The example above illustrates that we must consider the situation from the viewpoint of vessel A and vessel B simultaneously. Analogously to how we found the functions c_A°, $(t_1^A)^\circ$, $(t_2^A)^\circ$, and I_A° for vessel A, we can find the corresponding functions for vessel B. This is done below.

Quota Leasing Agreements from the Viewpoint of Vessel B.

Suppose that vessel A and vessel B make the quota leasing agreement (x_1, x_2). Depending on the following three cases,

$$\begin{aligned} &\text{Case I:} && \frac{Q_2^B - x_2}{Q_1^B - x_1} < \frac{l_B}{1 - l_B} \\ &\text{Case II:} && \frac{l_B}{1 - l_B} \leqslant \frac{Q_2^B - x_2}{Q_1^B - x_1} \leqslant \frac{u_B}{1 - u_B} \\ &\text{Case III:} && \frac{u_B}{1 - u_B} < \frac{Q_2^B - x_2}{Q_1^B - x_1} \end{aligned}$$

vessel B's optimal catch composition parameter, its optimal catch, and the associated maximal income are given by the following formulae.

$$c_B^\circ = \begin{cases} l_B & \text{(case I)} \\ \dfrac{Q_2^B - x_2}{Q_1^B - x_1 + Q_2^B - x_2} & \text{(case II)} \\ u_B & \text{(case III)} \end{cases}$$

$$((t_1^B)^\circ, (t_2^B)^\circ) = \begin{cases} \left(\dfrac{1 - l_B}{l_B}(Q_2^B - x_2), Q_2^B - x_2\right) & \text{(case I)} \\ \left(Q_1^B - x_1, Q_2^B - x_2\right) & \text{(case II)} \\ \left(Q_1^B - x_1, \dfrac{u_B}{1 - u_B}(Q_1^B - x_1)\right) & \text{(case III)} \end{cases}$$

$$I_B^\circ = \begin{cases} \left(p_1 \dfrac{1 - l_B}{l_B} + p_2\right)(Q_2^B - x_2) + q_1 x_1 + q_2 x_2 & \text{(case I)} \\ p_1(Q_1^B - x_1) + p_2(Q_2^B - x_2) + q_1 x_1 + q_2 x_2 & \text{(case II)} \\ \left(p_1 + p_2 \dfrac{u_B}{1 - u_B}\right)(Q_1^B - x_1) + q_1 x_1 + q_2 x_2 & \text{(case III)} \end{cases}$$

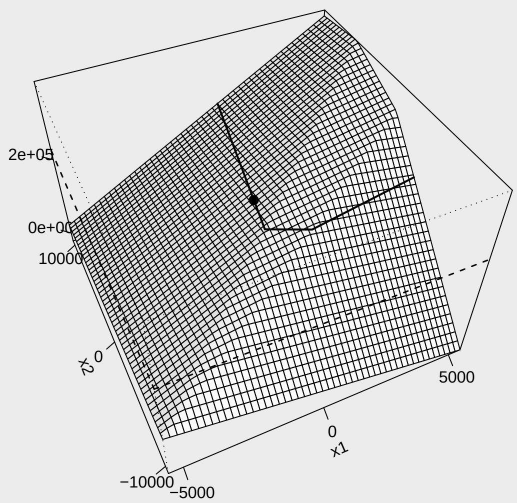
If vessels A and B decide not to exchange any quotas—corresponding to the quota leasing agreement $(x_1, x_2) = (0, 0)$—then their maximal incomes are $I_A^\circ(0, 0)$ and $I_B^\circ(0, 0)$, respectively. Obviously, vessel A is only interested in making a quota leasing agreement (x_1, x_2) if it is profitable, that is, if it results in a larger maximal income than $I^\circ_{\mathrm{A}}(0,0)$. Similarly for vessel B. Thus, the *profitable* quota leasing agreements for vessels A and B are the subsets of the square $[-Q_1^{\mathrm{A}}, Q_1^{\mathrm{B}}] \times [-Q_2^{\mathrm{A}}, Q_2^{\mathrm{B}}]$ given by

$$\mathcal{P}_{\mathrm{A}} = \{(x_1,x_2) \mid I^\circ_{\mathrm{A}}(x_1,x_2) \geqslant I^\circ_{\mathrm{A}}(0,0)\} \qquad \text{and} \qquad \mathcal{P}_{\mathrm{B}} = \{(x_1,x_2) \mid I^\circ_{\mathrm{B}}(x_1,x_2) \geqslant I^\circ_{\mathrm{B}}(0,0)\}.$$

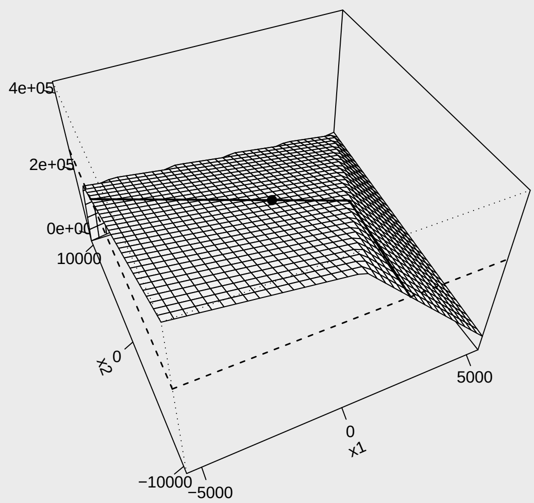
A necessary condition for vessels A and B to make a quota leasing agreement (x_1,x_2) is that this agreement is profitable for both parties, that is, (x_1,x_2) must belong to the intersection $\mathcal{P} = \mathcal{P}_{\mathrm{A}} \cap \mathcal{P}_{\mathrm{B}}$.

It is of interest for both vessels to find the set $\mathcal{P}$ of quota leasing agreements which are profitable for both of them. As illustrated by Example 2.6, there might not be any(!), however, usually there are many. In Example 2.3, vessel B is superior to vessel A in the sense that both vessels have the same number of quotas, but vessel B is more flexible in its catch composition (which ranges from 80%/20% to 20%/80%) than vessel A (which only ranges from 50%/50% to 40%/60%). However, even in this case there are many quota leasing agreements that are profitable for both vessels. This is explored in Examples 2.4 and 2.5 below.

Example 2.4 Consider the setup in Example 2.3. The graphs of vessel A's and vessel B's income functions $I^\circ_{\mathrm{A}}(x_1,x_2)$ and $I^\circ_{\mathrm{B}}(x_1,x_2)$ are illustrated below.

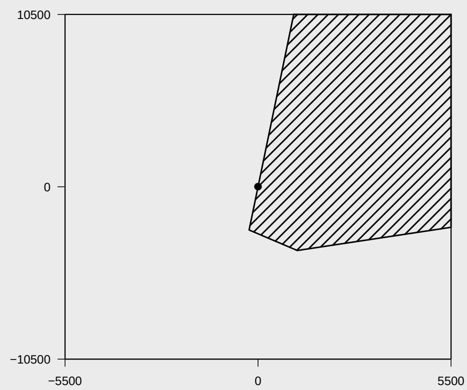


Graph of income function $I^\circ_{\mathrm{A}}(x_1,x_2)$ for vessel A

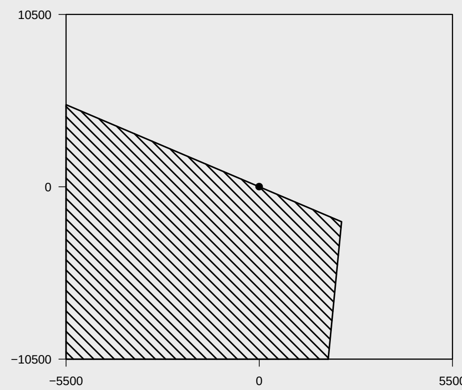


Graph of income function $I^\circ_{\mathrm{B}}(x_1,x_2)$ for vessel B

From these graphs one finds the sets of profitable quota leasing agreements for vessels A and B:

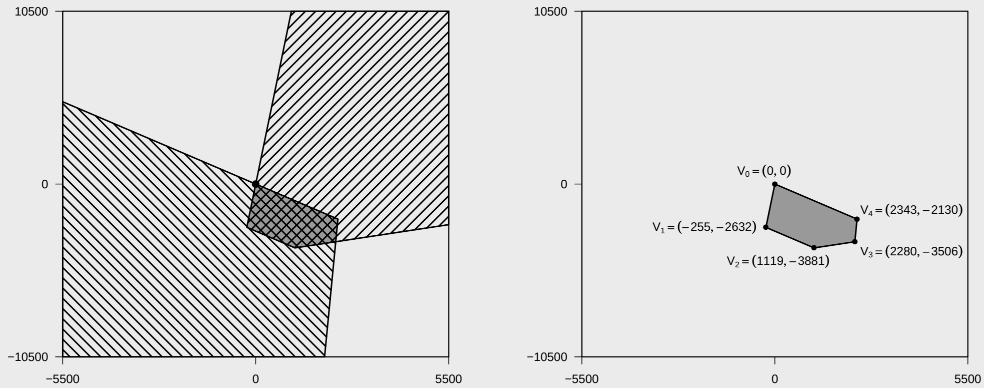


The set $\mathcal{P}_{\mathrm{A}}$ of profitable quota leasing agreements for vessel A



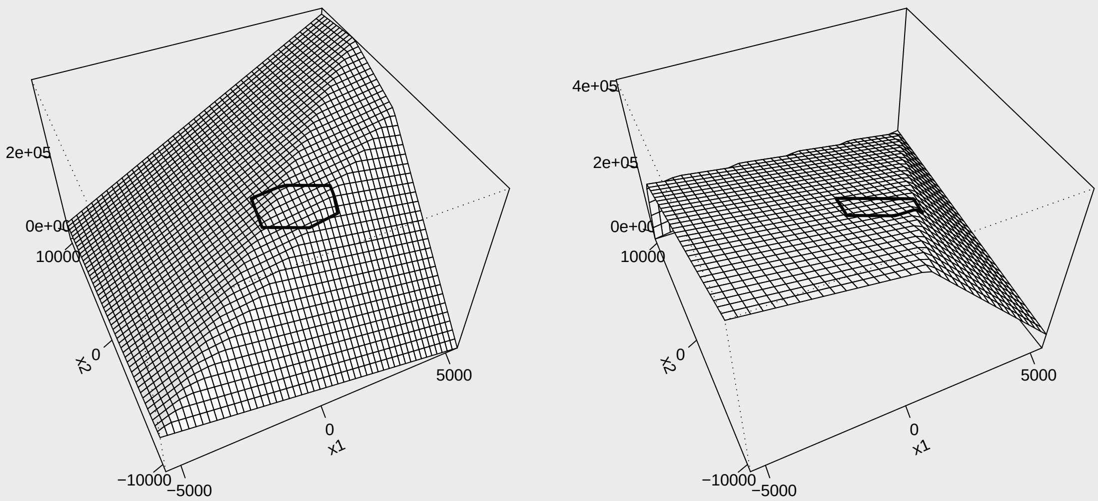
The set $\mathcal{P}_{\mathrm{B}}$ of profitable quota leasing agreements for vessel B

The set $\mathcal{P}$ of quota leasing agreements that are profitable for both vessels is found by intersecting the two sets $\mathcal{P}_\text{A}$ and $\mathcal{P}_\text{B}$, as illustrated below.



The set $\mathcal{P}$ of quota leasing agreements that are profitable for both vessels

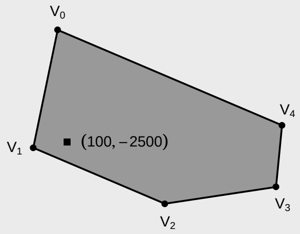
We see that $\mathcal{P}$ is a polygon with vertices $V_0, \ldots, V_4$ whose coordinates are shown in the figure above. The corresponding areas on vessel A's and B' income functions are:



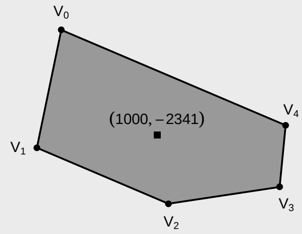
In general, the set $\mathcal{P}$ of quota leasing agreements that are profitable for both vessels is a polygon. Of course, it is possible to write down explicit formulae for the vertices $V_0, V_1, \ldots$ of this polygon in terms of the given market data p_1, p_2, q_1, q_2, and vessel data $Q_1^\text{A}, Q_2^\text{A}, l_\text{A}, u_\text{A}$ and $Q_1^\text{B}, Q_2^\text{B}, l_\text{B}, u_\text{B}$, however, since these formulae are rarther complicated, we shall not attempt to present them here. Instead, we note that in all specific examples, one can easily find the set $\mathcal{P}$, as it is done in Example 2.4.

Note that the quota leasing agreements (x_1, x_2) corresponding to *boundary points*, i.e. the edges, of the polygon $\mathcal{P}$ are not interesting, since either vessel A or vessel B will not make (or lose) any mony from such an agreement. Hence, the interesting quota leasing agreements correspond to *interior points* of the set $\mathcal{P}$.

Example 2.5 Consider the set $\mathcal{P}$ of quota leasing agreements that are profitable for both vessels, found in Example 2.4 above. The vertex $V_0 = (0,0)$ corresponds to the situation where the vessels lease no quotas from or to each other. We consider as well two other possible quota leasing agreements:



Quota leasing agreement $(x_1, x_2) = (100, -2500)$



Quota leasing agreement $(x_1, x_2) = (1000, -2341)$

Since $(x_1, x_2) = (100, -2500)$ (corresponding to the situation where vessel A leases 100 tonnes of cod quota *from*, and 2500 tonnes of haddock quota *to* vessel B) and $(x_1, x_2) = (1000, -2341)$ are points in $\mathcal{P}$, these quota leasing agreements are profitable for both vessels. However, as we shall see below, one vessel might benefit more than the other.

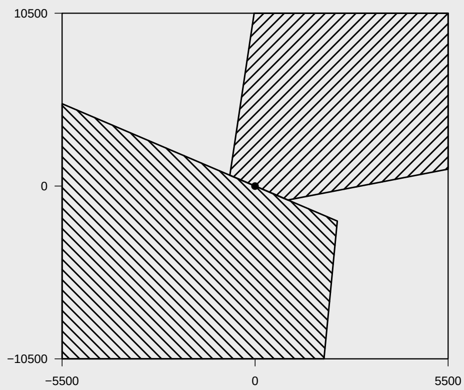
Computing the functions I_A°, I_B° and c_A°, c_B° at the points $(0,0)$, $(100, -2500)$, and $(1000, -2341)$ gives us the maximal income, and the catch composition required to obtain this, for each of the three situations.

(x_1, x_2)	Vessel A		Vessel B	
$(0,0)$	Income	$I_A^\circ = 225500$	Income	$I_B^\circ = 257000$
	Catch	$c_A^\circ = 0.60$ (40%/60%)	Catch	$c_B^\circ = 0.66$ (34%/66%)
$(100, -2500)$	Income	$I_A^\circ = 230500$	Income	$I_B^\circ = 283500$
	Catch	$c_A^\circ = 0.59$ (41%/59%)	Catch	$c_B^\circ = 0.71$ (29%/71%)
$(1000, -2341)$	Income	$I_A^\circ = 241250$	Income	$I_B^\circ = 272750$
	Catch	$c_A^\circ = 0.56$ (44%/56%)	Catch	$c_B^\circ = 0.74$ (26%/74%)

Compared to the situation $(0,0)$, where no quotas are leased, we see that if the quota leasing agreement $(100, -2500)$ is made then vessel A makes an additional 5000, whereas vessel B makes an additional 26500. If the quota leasing agreement $(1000, -2341)$ is made then both vessels make an additional 15750.

By the end of the day, whatever quota leasing agreement is made between the two vessels depends on which has the best negotiation skills.

Example 2.6 Consider the same setup as in Example 2.3, however, assume this time that vessel A's possible cod/haddock catch composition ranges from 40%/60% to 30%/70% (that is, $l_A = 0.6$ and $u_A = 0.7$). In this situation, the sets $\mathcal{P}_A$ and $\mathcal{P}_B$ have only an edge in commen, and hence there are *no* quota leasing agreements which are profitable for both vessels.



Annex

Boundary conditions for

Optimization model for the outtake of catch quotas under result based management

Prof. Henrik Holms paper is based on the boundary conditions described in this paper.

1. Optimizing the uptake of quotas under Catch Quota Management (CQM)

Basis

Catch-Quota Management (CQM) is a result based management with the following main features:

- The fisher is accountable for the total catch quota he has been given. All catches including by-catches must be covered by his quota and all catches of the given species, including eventual discards count on his quota.

- When the quota is exhausted the fisher must stop the fishery. If the quota for one species in a mixed fishery is exhausted fishing for all species in the fishery must stop. The fisher must optimize the utilization of his yearly vessel quota portfolio by
  i. Planning time and place of fishery or by using selective gear
  ii. Buying quota rights that covers his actual catches.

The input component is "the vessel" with a given use of fishing gear, fishing method and composition of species caught. Vessels with similar gear and fishing pattern belong to a "fleet". The main variables are

1. Fleet type. We operate with 3 fleets. Vessels of each fleet have a defined ability to control the catch composition within certain boundaries.
2. Fish catch-ability. In periods or areas fish may be more or less easy to catch. Here we consider catch-ability as an exogenous variable.
3. Quota availability. This depends on the allocation key between EU member states and on the changes in relative TAC's from year to year). We consider the availability as an exogenous variable.
4. Prices.

The output component to be optimized is tonnes of fish or value of fish in DKK

Problem

Establish or simulate the optimal outtake for given fleets against a given quota portfolio, a given typical quota allocation for the fleet and under the given boundary conditions.

Note on dynamics: The quota portfolio may change from one year to the other. Also the catch-ability may change as an overestimation of quotas may result in low catch ability and an underestimation in high catch ability, hence closing of a mixed fisheries where the high catch-ability do not match the underestimated quota

Boundary conditions

1st boundary condition

"The ability of the individual fleet to target the quota for the individual species in a mixed fishery. No change of vessel quota allocation is possible"

> Borderline case a) being that no targeting is possible in which case it is a simple (quota portfolio): (catch composition) calculation and borderline case b) that full targeting is possible in which case it cannot be considered as a mixed fishery.

Comment: The targeting ability of the fleets is defined under 3; Fleet structure and fisheries

2nd boundary condition

In certain member states it is possible to lease or buy quota rights. In that case fleets can buy rights to cover their need for quotas in mixed (or portfolio) fisheries). There is a limit to buying up: If there is only one fleet type buying up is only relevant to the point where all get the same portfolio, i.e. transferability only makes sense as long as the advantage for the receiving fleet exceed a possible disadvantage for the vessels transferring the fish. And of course transferability must take place within the total quota available for the member state.

Basis for simulation of quota utilization on account of vessel ability to target catch composition

Simulation and optimization for vessels in the following fleets:

I. Whitefish ("Scottish fleet") (cod and haddock)
   a. Typical catch composition for mixed fisheries
      haddock 50% : cod 50%
   b. Function regarding fleet ability to target individual species
      The vessel may influence the catch composition in the range between
      haddock 50% : cod 50% and haddock 60% : cod 40%:
   c. Available quota for the vessel: cod 11 tonnes and haddock 21 tonnes

II. Nephrops fishery ("Danish")
   a. Typical catch composition for mixed fisheries
      Nephrops 80 % : cod 20 %
   b. Function regarding fleet ability to target individual species
      The vessel may influence the catch composition in the range between
      nephrops 90% : cod 10% and nephrops 70% : cod 30%:
   c. Available quota for the vessel cod 4 tonnes; Nephrops 4 tonnes

III. Sole/plaice fishery ("Dutch")

   a. Typical catch composition for mixed fisheries

      Sole 20 % : plaice 80%.

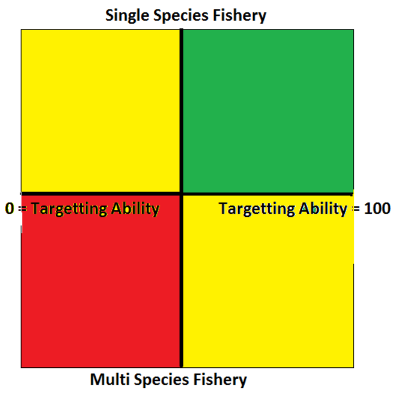
   b. Function regarding fleet ability to target individual species

      Sole 0 % : plaice 100%. Sole 20 % : plaice 80%.

   c. Available quota for the vessel: Sole 10 tonnes; Plaice 20 tonnes

Simulation of quota utilization on account of fleet targeting ability combined with quota transferability

Quota transferability increase quota utilization. In a system with transferability it is necessary to take account of the patterns of all relevant fleets in order to optimize on Member State level instead of fleet level. We may talk about 15-20 fleets with each their characteristics. For the sake of simplicity we consider for each of the above fleets only a "residual fleet" that encompass the average pattern of all other fleets.



The figure illustrate the comfort and discomfort zones in relation quota utilization on member state level (colours should be graded – if I knew how).

The effect of a management with transferability is that the y-axis moves to the left, leaving a larger green area

I. <u>Member state quota availability 11.000 tonnes of cod; 21.000 tonnes of haddock</u>

1. Whitefish (Scottish fleet) (cod and haddock)
   a. Starting quota 5.500 tonnes of cod and 10.500 tonnes of haddock
2. Residual fleet
   a. Starting quota 5.500 tonnes of cod and 10.500 tonnes of haddock
   b. Function regarding fleet ability to target individual species:

      The vessel may influence the catch composition in the range between haddock 20% : cod 80% and haddock 20% : cod 80%

II. <u>Member state quota availability 4.000 tonnes of cod; 4.000 tonnes of Nephrops</u>

1. Nephrops fishery (Danish)
   a. Starting quota cod 500 tonnes; Nephrops 3.500 tonnes
2. Residual fleet
   a. Starting quota cod 3.500 tonnes; Nephrops 500 tonnes

b. Function regarding fleet ability to target individual species

The vessel may influence the catch composition in the range between nephrops 90% : cod 10% and nephrops 0% : cod 100%:

III. Member state quota availability 10.000 tonnes of sole; 20.000 tonnes of Plaice

1. Sole/plaice fishery (Dutch)
   a. Starting quota: Sole 9.000 tonnes; plaice 18.000 tonnes
2. Residual fleet
   a. Starting quota: Sole 1.000 tonnes; plaice 2.000 tonnes
   b. Function regarding fleet ability to target individual species
      The vessel may influence the catch composition in the range between sole 10% : plaice 90% and sole 90% : plaice 10%:

Prices

Prices obtained for landed catches vary a lot. This is the suggestive levels pr kg: Cod 20 DKK; haddock 14 DKK; Nephrops 55 DKK; sole 85 DKK and plaice 10 DKK.

Prices for buying or leasing of quotas vary a lot too. Demand reflects i.a. to balance of available quotas compared to catchability. Landing prices and marginal value considerations (leasing a small quantity of a scarce species may release a large quantity of a plentiful species in a mixed fishery)

Buying of quotas is understood as permanent buying of quota shares and consequently the quota amount they release every year on account of TAC's being set. Buying typically takes place in context of structural changes as private scrapping investments in new vessels etc. Buying is not considered here. Leasing of quotas relate to the amount in tonnes of a given stock leased for the given quota year. Leasing may take place in context of planning the fishery for the entire quota year or it may relate to daily adaptation of vessel quotas to the development in the fishery – or to cover unforeseen bycatches. The leasing element is considered here as an important tool of flexible quota management. Prices pr. kg are set as follows: Cod 10 DKK; haddock 3 DKK; Nephrops 3 DKK; sole 6 DKK and plaice 2 DKK.